# Engineering Notes

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## Low-Speed Pressure Distribution on Axisymmetric Elliptic-Nosed Bodies

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### Introduction

BY taking the lead from early theoretical results, a semi-empirical representation of the pressure distribution on ellipsoids of revolution, and elliptic-nosed semi-infinite bodies of revolution in axial incompressible, potential flow has been constructed. The analytic expressions have been obtained by use of the peak velocity ratio and the normalized thickness dependent angle between body axis and radius vector, and are suited for rapid estimation of velocity and pressure distributions for nose slenderness ratios up to and including unity. With the exception of slender, semi-infinite bodies, the analytic representations should be quite satisfactory for preliminary design considerations.

### Analytic Expressions

#### Ellipsoids of Revolution

The pressure distribution on ellipsoids of revolution in axial incompressible flow was obtained from a solution by Maruhn[1] of the potential flow equations

$$C_p = 1 - A^2 \sin^2\theta = 1 - \left(\frac{V}{U_\infty}\right)^2_{\max} \sin^2\theta \tag{1}$$

where

$$\theta = \sin^{-1}\left(\frac{2\xi - \xi^2}{\delta^2 + (1-\delta^2)(2\xi - \xi^2)}\right)^{\frac{1}{2}} \tag{2}$$

and $\xi = x/a$, where $x$ is axial distance from the apex of the body. In the insert sketch in Fig. 1, $\theta$ is defined, and $a$ and $b = r$, the major axes of the ellipse, defining $\delta = b/a$, the nose slenderness ratio. The peak velocity ratio $(V/U_\infty)_{\max} = A$ is represented by the empirical expression

$$A = 1 + \frac{2}{3}\delta^{\frac{3}{2}} - \frac{1}{6}\delta^3 \tag{3}$$

$A$ vs $\delta$, according to Eq. (3), is shown in Fig. 2, and it is seen that the representation of the exact solution[1] for $\delta < 0.25$ and the numerical solution by Hess and Smith[2] for $0.2 \le \delta \le 1$ is very good.

Equation (1) reduces into the classic solution for the sphere

$$C_p = 1 - \frac{9}{4}\sin^2\theta \tag{4}$$

after inserting $\delta = 1$ in Eq. (3).

#### Elliptic-Nosed Semi-Infinite Bodies of Revolution

Combining the analytic form of Eq. (1) and the numerical results presented by Hess and Smith[2] for the velocity distribution on semi-infinite, elliptic-nosed bodies of revolution in incompressible flow, gives the pressure coefficient in the following form:

$$C_p = 1 - \left[A - \delta^{\frac{3}{2}}\frac{\tau - 1}{\tau}\left(\frac{1}{10} + \frac{1}{15}\chi^5\right)\right]^2 \chi \tag{5}$$

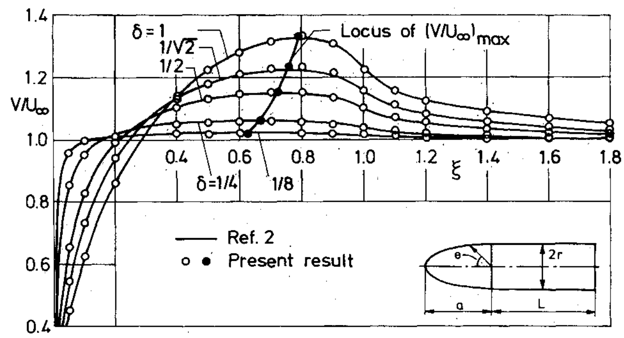


**Fig. 1 Representation of peak velocity ratio vs nose slenderness ratio for axisymmetric ellipsoids and semi-infinite elliptic-nosed bodies: a) slender bodies, b) slender to blunt bodies.**

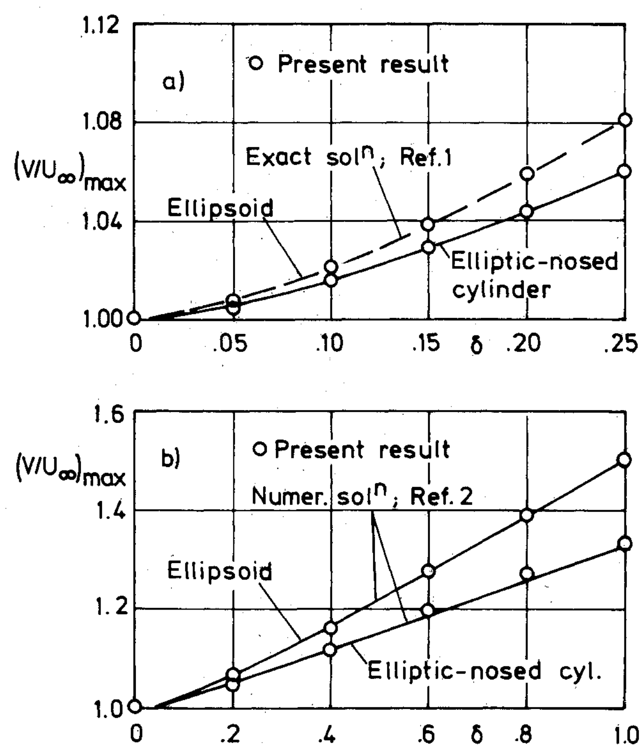


**Fig. 2 Representation of velocity distributions on semi-infinite axisymmetric bodies with elliptical noses.**

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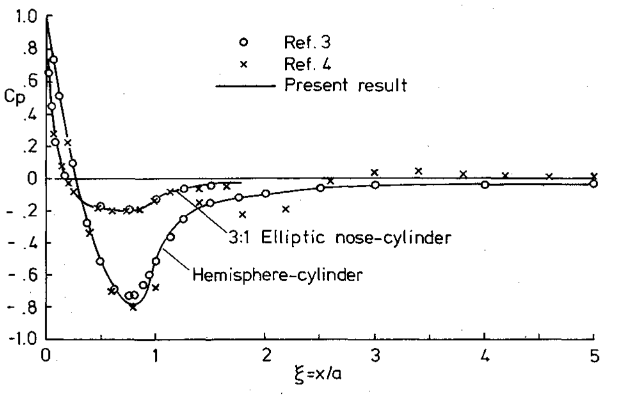


**Fig. 3 Pressure coefficient distributions on semi-infinite axisymmetric bodies with elliptical noses.**

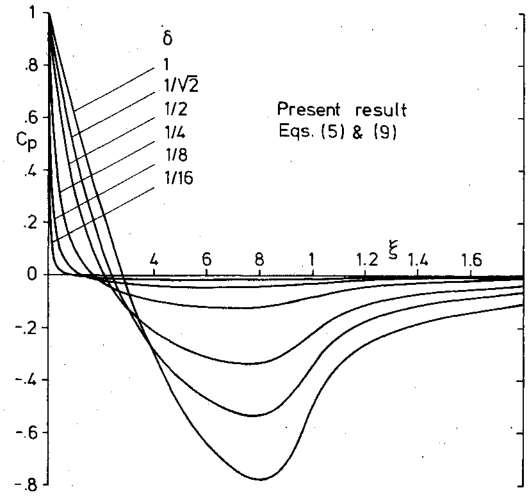


**Fig. 4 Comparison between pressure distributions obtained by distributed singularities and the present result.**

where

$$\chi = \frac{\sin^2\theta}{\sin^2\theta_{\max}}, \qquad \chi \le 1 \tag{6}$$

i.e., $\sin^2\theta$ has been normalized with respect to the length coordinate for the peak velocity ratio on the nose. This coordinate is

$$\xi_m = \xi_{(V/U_\infty)_{\max}} = \frac{1}{4}\tan^{-1}[(A_\infty - 1)3]^{\frac{2}{3}} + \frac{3\pi}{16} \tag{7}$$

where

$$A_\infty = 1 + \frac{1}{2}\delta^{\frac{3}{2}} - \frac{1}{6}\delta^3 \tag{8}$$

Equations (7) and (8) are empirical representations of the numerical results presented by Hess and Smith.[2] In Fig. 2b $(V/U_\infty)_{\max}$ vs $\delta$ is plotted, and it is seen that the result of Ref. 2 is overpredicted slightly for $0.5 < \delta < 1$. The factor $(\tau - 1)/\tau$ in Eq. (5) is unity for semi-infinite bodies and zero for ellipsoids, $\tau$ being the dimensionless afterbody length $\tau = L/a$ (see insert in Fig. 1). Therefore, Eq. (5) reduces into Eq. (1) for ellipsoids of revolution.

For $\xi > \xi_m$, where $\xi_m$ is defined by Eq. (7), the velocity distribution is represented by

$$\frac{V}{U_\infty} = (1 - C_p)^{\frac{1}{2}} = A_\infty - (A_\infty - 1)\psi^3[2 + \psi^3 + 2.5\delta(\psi - 1)^{\frac{3}{2}}]^{-1} \tag{9}$$

where

$$\psi = (\xi - \xi_m)(1 - \xi_m)^{-1}, \qquad \xi > \xi_m \tag{10}$$

The factor $(\tau - 1)/\tau$, present in Eq. (5) and being unity for semi-infinite bodies, is not shown in Eq. (9). For intermediate afterbody lengths $1 < \tau < \infty$, the factor is not known at present.

## Result and Discussion

The ability of the analytic expressions to represent theoretical velocity distributions on semi-infinite elliptic-nosed bodies of revolution in axial incompressible flow can be judged from Fig. 1, where a comparison is made with the results of Hess and Smith.[2] The comparison is made for several nose slenderness ratios, and it is seen that the correlation is satisfactory. The representation of the locus of peak velocity ratio according to Eq. (7) is satisfactory, also (the filled symbols). The pressure distributions corresponding to the above nose slenderness ratios are shown in Fig. 3. They represent a solution of the potential flow equations with the same accuracy as that seen in Fig. 1 for the velocity distributions.

The present representation of the numerical solution of the potential flow equations[2] can be used to test the accuracy of the results of other theoretical solutions. Among several existing solutions to the axisymmetric, blunted nose-cylinder flow problem by means of distributed singularities, only two recent results by Albone[3] and Christopher and Shaw[4] are chosen here for a comparison. The configurations are semi-infinite, axisymmetric bodies with a hemispherical and a 3 : 1 elliptical nose, respectively. The comparison is shown in Fig. 4. As the present result is a good representation for the potential flow solution,[2] it can be seen that the two numerical results[3,4] correlate very well with it in the apex region, but differ for the hemisphere-cylinder in the minimum pressure region[3] and on the afterbody.[4]

## Conclusion

Analytical representations of the result of solutions to the potential flow equations in axial, incompressible flow are presented for the pressure distribution on axisymmetric ellipsoids and semi-infinite, elliptic-nosed bodies of revolution. The correlation with theoretical solutions is satisfactory for preliminary design purposes.

## Acknowledgment

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